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From the archives

Editor's note: The following remarks are extracted from an article by Efraim Fischbein (1982), published in FLM3(2).

The concept of formal proof is completely outside the main stream of behavior. A formal proof offers an absolute guarantee to a mathematical statement. Even a single practical check is superfluous. This way of thinking, knowing and proving, basically contradicts the practical adaptive way of knowing which is permanently in search of additional confirmation. In principle, the formal structure of the adolescent's thinking possesses all the basic ingredients necessary for coping with both formal and empirical situations. Despite this, the current ways of trying and evaluating are mainly adapted to empirical contents. In solving a problem the mathematician proceeds, at the beginning, in the same way as the "empirical" scientist. He analyses the given situation, he tries to identify some general properties, some invariant relations or dependencies, *etc*. But at a certain moment this search process stops and a new situation appears: the mathematician has found a complete proof for his solution or theorem. Such a proof is the absolute guarantee of the universal validity of the theorem. *He believes in that validity.* This is a new situation in relation to natural mental behavior. Naturally, intuitively, we continue to believe in the usefulness of enlarging our field of research, of accumulating more confirmation. To think means to experiment mentally. Mental experience is the duplicate of the practical trial-and-error goal-oriented process. Therefore this ideal, the perfect proof, has no meaning for the natural empirical way of thinking. In order to really understand what a mathematical proof means the learner's mind must undergo a fundamental modification.

Of course he can learn proofs and he can learn the general notion of a proof. But our research has shown that this is not enough. A profound modification is required. A new completely non-natural "basis of belief" is necessary, which is different from the way in which an empirical "basis of belief" is formed. *The concept of formal, noninductive, non-intuitive, non-empirical proof can become an effective instrument for the reasoning process if, and only if, it gets itself the qualities required by adaptive empirical behavior!*

In other terms: *formal ways of thinking and proving can liberate themselves from the constraints of empirical knowledge if they become able to include in themselves those qualities which confer on the empirical search its specific productivity.* We refer to the global, synthetic, intuitive forms of guessing and interpreting.

It is not enough for the pupil to learn formally what a complete, formal proof means in order to be ready to take complete advantage of that knowledge (in a mathematical reasoning activity). A new "basis of belief", a new intuitive approach, must be elaborated which will enable the pupil not only to understand a formal proof but also *to believe* (fully, sympathetically, intuitively) in the *a priori* universality of the theorem guaranteed by the respective proof. As in every form of thinking, we need, in addition to the conceptual, logical schemas, that capacity for sympathetic, direct, global acceptance which is expressed in an intuitive approach. After learning a formal proof we have to reach not only a formal conviction—but also the internal direct agreement which tells us: "Oh yes. It is obvious that the described property *must* be present in every object which belongs to the given category." *The feeling of the universal necessity of a certain property is not reducible to a pure conceptual format. It is a feeling of agreement, a basis of belief, an intuition—but which is congruent with the corresponding formal acceptance.*

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